DISCOVERING HIDDEN TRANSFORMATIONS

MAKING SCIENCE AND OTHER COURSES MORE LEARNABLE

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Abstract. Problem-solving research has consistently found that students have great difficulty when asked to apply concepts and principles discussed in texts and lectures. It is often unnoticed knowledge extensions or transformations that allow faculty experts to solve such problems. Because faculty members normally use these extensions automatically, they do not emphasize them in their teaching. Therefore, knowledge transformations often remain completely hidden to students. This paper presents a first attempt at a field guide to help faculty members identify and teach about hidden knowledge transformations. Examples from introductory psychology and physics teaching are included, along with some pedagogical techniques that have been found to be helpful in dealing with these in both disciplines.

When watching a video that made the thoughtful, individual steps in solving a physics problem very clear, one of my computer science colleagues blurted out, "Hey, that's unfair! It's almost as if that video is showing students what's going on in my mind. Where will the challenge go?" Many college faculty feel that it is almost cheating to show students how to do real science problems—those that involve application—in a form and at a pace they can understand. My friend was somewhat facetiously suggesting that showing others the problem-solving "tricks" he figured out on his own takes away exactly that quality that allowed him to stand out enough in the natural sciences to make him special. This was making available to the typical college student a skill in which this professor excelled. This transparent approach took away his "elite" status. Another colleague suggested that sometimes we seem to be working a lot harder at demonstrating how clever we are rather than helping students become competent in the field. This is not effective teaching for any group of students, and a shift in both our approach and attitude is required.

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A primary thread in my own teaching career has been searching for ways to help students learn to apply what they are taught. The application of new information is a demonstration of mastery and, in the end, one of the most useful products of what we teach. But I have only gradually understood the complexity of the task of applying ideas in new contexts.

I teach psychology. As a teacher, I want my students to use their knowledge in contexts other than the ones in which they are learned. In Bloom et al.'s *Taxonomy of Educational Objectives: The Classification of Educational Goals, Handbook I: Cognitive Domain* (1956), this would be at the application level, and in Anderson and Krathwohl's revision (2001) of Bloom's taxonomy, this is classified as "apply-implementing . . . when a student selects and uses a procedure to perform an unfamiliar task" (78). When I first started teaching, only about 10–20 percent of my students could solve the application problems posed to them.

In retrospect, I wish that I had not been so surprised at this. When doing psychotherapy, I found that the most difficult and time-consuming aspect is helping patients apply insights from therapy to their daily lives. As a therapist, I am very attentive to the problems, frustrations, and difficulties that my clients have in applying their learning. In teaching, however, I often have spent all of my class time presenting concepts while expecting my students to apply them on their own. Indeed, I sometimes gave my students their first exposure to application problems on tests. I have since realized that when I do this, I leave students the most difficult learning task with no guidance or appropriate encouragement.

It was reassuring to find out I was not alone in this practice. Allyn Jackson (1989) described some of the reasons why he and Uri Treisman developed an innovative approach to calculus teaching. Jackson pointed out that students taking college-level calculus

> are often shocked to find that "cookbook" methods of problem solving and rote memorization no longer suffice. And while some of the textbooks used in Berkeley calculus classes emphasize rote learning and a formulaic approach, the examinations, usually prepared by the professor, typically require a higher level of mastery. (Jackson 1989, 26)

My review of curriculum innovation in the natural sciences has led me to think that my concerns about student difficulties with application learning might be relevant to those who are involved in education more generally. In particular, hidden versions of what I call "knowledge transformations" may play a large role in making science and other college classrooms "chilly" and unnecessarily difficult for everyone but those with the most advanced preparation.

To change our behavior, we must be aware of our actions. The classification scheme introduced in this paper is meant as a first attempt to develop a field guide for locating and correcting places in our courses where we unintentionally make the courses more difficult for students. This is a catalogue of the kind of knowledge transformations that we expect students to make on a regular basis. It is not an exhaustive list; rather, it is a first set, or a beginning, that is designed to help faculty members think about the complexity of the cognitive tasks we take for granted and expect students to perform with little instruction and guidance from us.

What Is a Knowledge Transformation?

In brief, a knowledge transformation is a process that alters a concept or principle from the form in which it was specifically presented to any other form. Moving from a memorized definition of some concept to recognizing an actual example usually requires a sophisticated set of knowledge transformations. An example from my introductory psychology course may clarify what I mean. In that class, I ask two different types of exam questions:

1. A simple recall type of question: Which of Freud's defense mechanisms makes unacceptable impulses look like their opposite?

   a. reaction formation
   b. projection
   c. displacement
   d. rationalization

Answer: a

2. An application of the understanding of the concept's meaning: As her parents became increasingly more abusive toward her, Sharon began to express emphatically her great admiration for her parents with apparent sincerity.

   Sharon's behavior illustrates most clearly the defense mechanism of:

   a. reaction formation
   b. projection
   c. displacement
   d. rationalization

Answer: a

The first example is no more than a definition of a term from a textbook or lecture. The student merely has to recognize the material in the same form as it was previously presented. This requires no change in the form of the knowledge—the input form to the student matches exactly the output form required later.

The second question requires recognition of an example of the concept in a new context or situation. This application question requires that students transform the knowledge or definition from the text by means of a remarkable number of operations or steps. First, they must infer that Sharon's unconscious reaction to her parents' abuse is likely to be strong anger. Second, it is necessary to know that such strong negative reactions to parents are socially unacceptable. Third, they must realize that the opposite reaction, "admiration," is more socially acceptable. Finally, they must remember that the defense mechanism that turns unacceptable impulses into their more socially acceptable opposite is called "reaction formation." An application of this concept requires a complex and elaborate set of changes in the form of the student's knowledge. (Because it requires students to take the definition and see how it operates in an applied example, I will label this a "procedural transformation" below). I do all of these transformations automatically without reflection or awareness.

Examples such as these are available for every discipline taught at the college level. Once we recognize the complex set of inferences and extensions of knowledge that we accomplish automatically but that are required to answer the second question correctly, it is no longer so surprising when most students answer incorrectly. And, as many readers will agree, the second question is much more likely to be the version encountered on an examination. As faculty, we feel that these exercises, heavy with transformation, are strong probes of the students' understanding. From the students' perspective, however, they may seem more like tricks to trip them up.

A Classification Scheme of Knowledge Transformations

Soon after examining this example, I began to see other kinds of knowledge transformations that I expected students to make. As I discussed these ideas with a colleague, Craig Nelson (1989), he suggested I read a book by Arnold Arons, *A Guide to Introductory Physics Teaching* (1990). The content of this book is also contained in Arons's *Teaching Introductory Physics* (1997), along with several other valuable sections. As I read Arons's list of key underpinnings necessary for students to understand any introductory physics course, I could see that he broke up the learning into small enough steps so that students were explicitly taught knowledge transformations that were so often skipped over by other physics instructors. Similarities between what he discussed and the knowledge transformations that I expected my psychology students to make were apparent. If these ideas made some sense of the cognitive problems that students encountered in two such widely differing fields as psychology and physics, I thought other disciplines might be able to use these as a heuristic aid in finding how knowledge transformations applied to their teaching as well. It may even be that facility with these knowledge transformations is a significant part of the difference between a novice and an expert in many fields (Anderson 1987, 1993; Kurfiss 1988; Medin, Ross, and Markman 2001).

Summarized in table 1 for each transformation type are examples that I found in physics and their parallels from psychology. The physics examples are taken from the first chapter of Arons's books on teaching introductory physics (Arons 1990, 1997).

1. *Procedural Transformation*: Arons starts with the concept of "area," which he says is a foundation concept that students must understand before they can master beginning physics. He mentions that students often enter physics courses with the rote response of "area equals length times width" (1990, 1; 1997, 2). To find the area of a rectangle, students must convert this memorized sentence into a procedure or a set of operations. They must measure length and width and multiply the results. This is similar to the distinction that cognitive psychologist John Anderson makes between declarative knowledge and procedural representation as one of the first things that novices must learn to do to become experts (1987, 1993).

In the example above, in which psychology students are expected to go from the rote memory of the definition of "reaction formation" to identifying an applied example of it, they must make a procedural transformation. Students must have a procedure for identifying the key features of the defense mechanism embedded in the example.

2. *Conceptual Transformation*: Arons would like students to transform their concept of area to an operational definition that they can use with irregular figures (ones that do not have an easily discerned

TABLE 1. List of Transformation Types

Transformation type	Illustration from Arons's introductory physics	Introductory psychology illustration
1. PROCEDURAL—Transforming knowledge so that an abstract concept can be converted into a procedure that can be used in a concrete situation.	Teaching students to go from the rote or declarative memory of "area equals length times width" to actually measuring a rectangle and computing the area.	Teaching students to go from the rote memory of the definition of "reaction formation" to identifying an example.
2. CONCEPTUAL—Abstracting more general principles from declarative (rote) or procedural knowledge.	Teaching students to change the level of abstraction of their definition of area to something like "selecting a unit square, imposing a grid on a figure, and counting the squares within it."	Teaching students to go from definitions of a specific defense mechanism to an understanding how all of them are derived forms of "repression."
3. CONTEXTUAL—Transforming knowledge so that a concept or procedure can be used on a problem embedded in a new situation.	Asking students to find the area under a graphed curve after learning to deal with areas of irregular figures.	Asking students to use "reaction formation" sequentially in combination with other defense mechanisms to modify a single instinctual impulse (in a more "real life" situation).
4. ANALOGICAL—Locating the likeness between two concepts or operations.	Asking students to use their understanding of "area" to comprehend "volume."	After students understand Freud's idea of "defense mechanism," using their intuition to comprehend Rogers's idea of "distorted symbolization" of experience.
5. SYMBOLIC—Using symbols to represent relations.	Translating English sentences into algebraic equations. For example, once students can compute "densities" when given "volumes" and "masses," then using "densities" and "masses" to solve for "volumes."	In psychology, symbols are used in diagrams for classical conditioning: Before: NS → No Response US → UCR During NS → US → UCR After CS → CR
6. METAPHORICAL—Using one kind of symbol system to stand for or represent a concept that was originally expressed in a different symbol system.	After understanding ratios in numbers and formulas, moving to "graphical representations." Newtonian mechanics require that students use a "frictionless world" (quite different from anything they have experienced) as a metaphor to understand motion.	Moving from a verbal chart of the operation of a defense mechanism to doing a flow diagram of the same processes.

(table continues)

TABLE 1. (*Continued*)

Transformation type	Illustration from Arons's introductory physics	Introductory psychology illustration
7. ARBITRARY—Transformations that are often fixed by the history of a field and therefore have little obvious rational basis; they are often extremely hard for students because faculty take them for granted and do not even notice the students' problem.	Using the letter "v" to stand for "instantaneous velocity and change in instantaneous velocity" without informing the students of the switch. Using a common word such as "force" in an uncommon way in physics.	Freud uses the word "sex" to include a much wider range of pleasurable experiences than we refer to in everyday language (others such as "anxiety," "depression," "schizophrenia," and "multiple personality" also have different, more precise definitions in psychology).

length and width), such as finding the area of an inkblot. According to Arons, "many students faced with this problem have very little response of any kind" (1990, 2; 1997, 2). Part of the reason that students may have no response is that they must go through a number of complex steps to attack it. They must move from concrete operations, such as measuring length and width and multiplying, to a much more abstract conceptualization of "finding area." The concept now must include three different steps: (1) select a unit square; (2) impose a grid of unit squares on the figure; and (3) count the squares (and fractions) within the figure.

Because students are asked to make such a significant change in their definition of area, a shift that requires them to move one significant step up and two significant steps down the ladder of abstraction, I think this might be seen as a conceptual transformation. Looked at this way, it is not surprising that many students will be mystified by this task. Mystification can rapidly turn to discouragement when the student recognizes that the instructor and some other students behave as if this is or should be an obvious extension.

In psychology, when we try to get students to see that all defense mechanisms are derived forms of "repression," we are asking them to develop a much more abstract conceptualization. To do this, they must see that Freud was trying to account for all of the different ways that animal instinctual impulses could be modified to be more socially acceptable drives and urges. Repression was the first way he saw that unacceptable sexual wishes could be managed, and he later discovered that there were a number of other ways in which the mind could modify these urges.

3. *Contextual Transformation*: In contextual transformations, students are asked to apply their knowledge in a new context. When Arons asks students to use their ability to measure the area of irregular figures to find the area under curves on graphs, he is requiring a contextual transformation. Unlike a conceptual transformation, the contextual change involves a simple extension in one dimension. An inkblot is an object defined on all sides by an outline. A graph of a function is different only in that it is not bounded by drawn, explicit outlines at the left and right "edges." However, we frequently do not teach how we are thinking about those unbounded edges.

In psychology, we might ask the students to see how "reaction formation" can be used sequentially in combination with other defense mechanisms to modify a single instinctual impulse in a different situation. Here is an example of a question that requires contextual transformation:

> Although her parents seemed overly controlling toward her, Sharon emphatically expressed her closeness to her parents as the main reason she was unwilling to take a better-paying job in a nearby town where her boyfriend lives. Sharon's behavior illustrates sequential use of the defense mechanisms of:
>
> a. reaction formation followed by rationalization
> b. projection followed by reaction formation
> c. displacement followed by rationalization
> d. rationalization followed by reaction formation
>
> Answer: a

Although these types of transformations are usually obvious to instructors, I often find that less than 20 percent of my students are able to make these transitions on the first try. Cognitive psychologists in research situations find that without hints and special learning techniques, only about 10 percent of students solve this kind of problem (Gick and Holyoak 1980). They find that even minimal changes of "surface" characteristics can be quite confusing for students. To solve this kind of problem requires that students attend to the "deep structure" of a problem.

4. *Analogical Transformation*: Similar to the contextual transformations, analogical ones are an extension of the concept but are significantly more cognitively complex. In this transformation, the student must use the similarity between a concept they understand and a more complex concept that they are learning. For example, when Arons wants students to use their understanding of "area" as an analogy to understand "volume," he asks them to make an analogical transformation. This involves a true analogy because "area" in two dimensions is in many ways analogous to "volume" in three dimensions. Both of these concepts require students to use the idea of "ratios" and space filling.

In psychology, a similar problem might occur when we teach theories of personality other than Freud's. One theory we usually compare with Freud in introductory courses is Rogers's "client-centered," or humanistic, psychology. Rogers's idea of "distorted symbolization of experience" is an analogue for Freud's defense mechanisms (1951, 500). Rogers wants to explain emotional disturbance without using the idea of the unconscious. Instead of having a defense mechanism that operates in the unconscious between the animal instincts and conscious experience, he has distortion occurring as we interpret our responses to situations. To use the previous example, Sharon's positive feelings toward her parents would be seen in Rogers's theory as a "distorted symbolization" of her inner experience. In particular, she could be misinterpreting her feelings of anger and guilt when criticized by her parents as feelings of admiration for them. (She might thus be in a position of believing she felt admiration for anyone who criticized her and made her feel angry and guilty.) This is a "distorted symbolization" in Rogers's system, and it has a function analogous to the defense mechanism of reaction formation in Freud's system. In fact, Rogers's single concept of distorted symbolization is used to account for the operation of most of the defense mechanisms in Freud's system. We tend to explain this once and assume that students will be able to use the analogy to explain the concepts of rationalization, projection, displacement, sublimation, and so on, but students often do not see this nearly as easily as we expect. In fact, only by going over examples of each of these and helping them understand how each can be seen as a misinterpretation of inner feelings can we help students truly see these as directly analogous to Freud's concepts. Similar analogies are made for the constructs in the other major personality theories, and we rarely help students through successive specific examples.

5. *Symbolic Transformation*: These manipulations and extensions are also analogical transformations, but they are so ubiquitous and important that they seem to deserve a category of their own. For example, Arons gives physics students problems such as, "[G]iven the amount one paid for 1 kg of material, use the *inverse* to figure how much of the material you would get for $1" (1990, 5; 1997, 6). These involve the substitution of letters for numbers or variables. Arons notes that many science students have trouble transforming simple English sentences into algebraic form (1990, 14–15; 1997, 17–18). He reports research by Clement, Lochhead, and Monk (1981), who gave problems such as:

> Write an equation using the variables ***S*** and ***P*** to represent the following statement: "There are six times as many students as professors at this university." Use ***S*** for the number of students and ***P*** for the number of professors. [Before going further, the reader might like to write down an answer.] (Arons 1990, 14–15; 1997, 17–18)

This problem was missed by 37 percent of calculus students and 57 percent of nonscience majors. Most wrote 6S = P. Note that this is a direct or superficial word-to-symbol translation. Conceptually, however, the equation implies that if you have the number of students and multiply it by six, that quantity is equal to the number of professors. This is quite different from having six times more students than professors. The translation from words to symbols is conceptually flawed. As Clement, Lochhead, and Monk (1981) conclude, "what makes teaching (and learning) of these translation skills so difficult is that behind them there are many unarticulated mental processes that guide one in constructing a new equation on paper" (289).

When students learn how to use symbol systems, it greatly increases their ability to apply the concepts in new situations. However, much less symbolism is used in psychology, and it is not nearly as well developed or as powerful as in physics and other natural sciences. In psychology, for instance, symbols are used in diagrams for classical conditioning. Conditioning starts with a neutral stimulus (NS), such as playing defense in a basketball game. The person has no special response to this stimulus. An unconditioned stimulus (US)—for example, a coach yelling at a player—leads to an automatic unconditioned response (UCR): being scared. During conditioning, a player is on defense (neutral stimulus); this is paired with the coach yelling at the player (the unconditioned stimulus), who then shows and feels fear (the unconditioned response). If this is done a number of times, then learning or conditioning occurs that is reflected in the "after" diagram below. Playing defense (NS) now becomes a conditioned stimulus (CS), and the player feels very alert, which is a conditioned response (CR). Because the conditioned response (very alert) is usually lower in magnitude than the unconditioned response (fear), it is given a different symbol.

There are also standard diagrams for operant conditioning and a few other processes in every introductory text in psychology (see figure 1). It took me some time to realize how much practice students need to understand these systems deeply enough to apply them.

6. *Metaphorical Transformation*: This involves a more dramatic symbol system shift than the algebraic and analogical transformations (on a smaller scale, this is perhaps equivalent to a paradigm shift in a discipline). In physics, relationships expressed in formulas are represented as curves in coordinate systems. Although this shift in representational system may be obvious to experts in the field, it can be quite mystifying to novices. Such transformations may be particularly significant in physics, as students are often required to translate ideas from verbal to pictorial to mathematical and graphical forms, as well as moving back and forth from concrete problems to abstract principles and ideas.

Reading and discussing these metaphorical transformations with physics colleagues led me to realize not only how important these are for understanding our disciplines but also how dramatic are the leaps of metaphor that we require. To understand most of the Newtonian laws of motion, for example, requires that students imagine and think quite deeply about a "frictionless" world. I remember being told about this as if it was both easy and quite obvious to achieve, yet I had absolutely no life experience that was even close. Perhaps understanding would be easier if we imagined something more akin to the realm of science fiction than the practical world of careful observation. A frictionless world does not exist, except in

Before	*During*	*After*
NS → No response *US → UCR*	*NS → US → UCR*	*CS → CR*

FIGURE 1. Typical classical conditioning diagram.

science fiction (there are several demonstrations that can give students an experience something very close to a frictionless world, such as using a large block of dry ice on a glass surface or wet ice on a surface of soap flakes). It is essential for solving problems of mechanics, but the leap of imagination required is quite dramatic. From a transformational point of view, it is little wonder that more than a few natural science majors fail to move from their common sense, "Aristotelian," or medievalist views of reality by the end of their first few physics courses.

My students had similar difficulty when I went from a verbal description of the operation of a defense mechanism to doing a flow diagram of the same processes. Freud used metaphors of energy flows when describing these processes, so I thought it might be helpful to do flow diagrams using vectors to illustrate the various conflicting mental energies. Many of my physics and pre-engineering students reacted with, "Oh, now I see!" but many of my other students were totally confused. The flow charts were moved to extra credit items on exams, and verbal charts were developed for the majority of students. Had I been thinking in terms of the metaphorical transformation required by this process, I would have been much more careful to make both verbal and visual presentations of the ideas available to students at the beginning.

7. *Arbitrary Transformation*: More difficult for students than metaphorical transformations are what I call arbitrary transformations. Arons points out that "there are still some authors who seem to think life is made 'easier' for students by introducing acceleration a = v/t, apparently failing to realize the confusion caused by using the same symbol v for either an instantaneous velocity or a change in instantaneous velocity" (1990, 28; 1997, 32). These changes are so cryptic for students that they could readily be called unfair transformations.

Although arbitrary transformations seem to be the simplest conceptual leap from a faculty perspective, they can be stunningly confusing to the uninitiated. This is especially a problem when we want students to learn a new definition for a word already in their vocabulary. In physics, Arons (1990, 1997) elegantly describes how the word "force" is introduced by association with "muscle sense" and then is "redefined" to include inanimate objects that can impart acceleration to other objects and then is expanded even further, as in an example of Newton's third law: "The Earth exerts a downward force on a person and the person exerts an equal and opposite force on the Earth" (1990, 296; 1997, 354).

In my terms, the concept is dramatically transformed and often done so quickly that it appears hidden from students. In psychology, the word "sex" can mean gender or intercourse or any aspect—physical, biological, or cultural—of either of the two. Freud used the word "sex" to include a very wide range of pleasurable experiences that we refer to in everyday language, such as the "organ pleasure" involved in eating or kissing. Differences in definition are often subtle, complex, and counterintuitive. It is no wonder that students often have great difficulty learning how these new concepts fit with old words. Other words, such as "anxiety," "depression," "schizophrenia," and "multiple personality," also have different and more precise definitions in psychology and can cause difficulties for most students.

Implications of the Higher-Level Transformations

It may be tempting to argue ad infinitum the precise definitions (to define the "edges") of each of the described transformations. However, we may be better served to understand that in each of our disciplines, many of these transformations are crucial to demonstrating successful mastery. In every classroom, there are many students for whom these transformations do not come naturally or for whom this type of transformational thinking has never been explicitly taught. It should be noted that students who go to a good college preparatory school, have a strong set of Advanced Placement science courses, or are deeply involved in an enriched math-science center get a year or more of extra exposure to learn about these transformations. Students who have not had such opportunities are at a distinct but, I believe, surmountable disadvantage. As we become more aware of our own invisible processes of thought, we can work to make these transformations explicit and accessible to our students.

The most rapid and complex set of metaphorical and other high-level transformations that I have experienced was in a mathematics class in multivariable calculus. In less than one hour, the instructor made an uncountable number of metaphorical shifts. He showed students how second derivatives can be used to test for "local maximums and minimums" in a function. This required defining each concept (1) linguistically, then translating each into (2) mathematical notations so that symbolic transformations could be made (deriving theorems), then creating functions to exemplify these expressions, drawing (3) pictures of three-dimensional surfaces that were illustrations of these functions, and making (4) numerical calculations to locate points on these surfaces. He went through about six theorems in one class period and was even able to use a computer to generate some of the surfaces. Throughout the class period, he rapidly went back and forth between these four modalities of thought as if he were putting students through a mental workout equivalent to the training routine of a team of Olympic-level gymnasts.

When I asked this master teacher for his opinion of the percent of the class that he expected to follow him through this taxing exercise, he estimated that no more than one-third of the students could do so. One wonders what is expected of the other two-thirds of his students. If his goal was to sort or weed out students from future courses, then he was doing an effective job. I would not be sure, however, that he was weeding out only students of lesser ability. He was weeding out students who were not familiar or quick enough with the particular knowledge transformations he was using. There may have been many more students who could have followed his performance if he had made his transformations more explicit.

Although most of us take for granted our students' skill in following and being able to reproduce these operations, there is strong reason to believe that they do not have these skills as they begin our courses. Most important, from a psychological perspective, even the easiest of these transformations can be "hidden" or mystifying to students. They are often obvious to a physicist, psychologist, mathematician, or chemist (experts) who use them almost daily. We easily forget that students (novices) typically are not clued in to them. In fact, I would argue that most students react with confusion and mystification each time they encounter most of these transformations. The speed with which discouragement appears in the student is variable but perhaps inevitable under these conditions.

Procedures for Deepening Learning

In my thirty years of teaching, I found that only 10–20 percent of my students can perform the transformations that I expect on my application-level test items unless I take special measures. When I take the time to work through the process of applying ideas at a reasonable pace and give students a lot of practice in different contexts, I can get a much greater percentage to perform at this level.

In "intensive explicit transformational practice," my most effective procedure, I group students into cooperative work teams of four and give them progressively more difficult problems in class (see Grossman 1994 for a more detailed description). I use lectures to present new concepts and usually demonstrate how they apply to at least one case example. Then, in recitation sessions with the work teams, I serve as a "guide on the side" to help the teams learn the transformations required to apply a concept in a variety of different problem situations.

In these sessions, it is very useful to find two or three groups that have different answers to a problem. I usually have them write their answers on the board and explain how they arrived at their solution. We then discuss the relative merits of the various answers. It is important that the students articulate their reasoning processes. In the end, students can usually see which answers are the better ones. I always take time to point out what is correct and creative in each of the answers but go on to describe how such a problem will be graded on the next test. During these discussions, I often recognize the transformations that I have not noticed but that are necessary for students to solve the problems. We often have to go back and revise our notes from the lecture to include elaborations on the concepts being taught.

A second type of assignment that helps students achieve this level of learning is the application paper. I ask students to write a short paper (one to three pages) every other week applying at least one of the concepts they have learned in the course to some aspect of their lives. There are three crucial aspects of this assignment. First, they must give a detailed account of their life experience. Students tend to write in vague generalities about their lives rather than describing what they see and hear. It often takes one or two communications and rewrites before all students in the class can give me enough sensory detail in their descriptions to be useful for application. I give models of good and bad reports, but many students need to get direct critical feedback and a chance to rewrite their work before they get the level of detail necessary to enable accurate applications. Second, they need to write out definitions of the concepts from the text or lecture that they are going to apply to their experience. I find that if they do not type out the exact definition, they frequently leave out the most important qualifying phrases. Third, I ask them to do a point-by-point comparison between the definition and their experience, one in which they point out differences and similarities. Again, if I do not ask them to be specific about this, they will often make an inaccurate application. This assignment gives me an opportunity to see how well they apply concepts. It also often provides the opportunity to discover hidden transformations that make the assignment difficult for students. By means of intensive transformational practice, I find that I can get 80–90 percent of my students to apply concepts successfully in new contexts on their tests.

It is interesting to see that the special teaching methods Arons has developed parallel my intensive explicit transformational practice. He also has discovered how important it is to give students lots of practice making knowledge transformations. He makes learning concepts at a deeper level a prominent objective. He calls one of his main techniques "exercises in verbal interpretation" (Arons 1990, 4; 1997, 5). This involves asking students to put into their own words (make explicit) every important concept and then practicing each concept in several different contexts. As he says,

> Without such practice in at least several different contexts, students do not think about the meaning of calculations they are expected to carry out, and they take refuge in memorizing patterns and procedures of calculation, manipulating formulas, rather than penetrating to an understanding of the reasoning. As a consequence, when they find themselves outside the memorized situations, they are unable to solve problems that involve successive steps of arithmetical reasoning. (1990, 4; 1997, 5)

Hake (1991), a self-described "convert of Arons," points out that this is only a portion of Arons's approach. In fact, he gave talks in which he listed eleven salient features of the "Arons-advocated method." He suggested three articles for those who would like more details about his take on Arons's approach (Hake 1991, 1992, 2002).

Arons and I agree that a main gauge of knowledge competency or scientific literacy is application-level learning; students must be able to use concepts in different contexts and forms than the ones in which they were taught. In my terms, his test for whether students have learned concepts is their facility in using knowledge in "transformed" ways. I have found that categorizing the transformations makes them easier to see and thereby helps me teach students to do them.

Research by cognitive psychologists on problem solving offers some interesting support for these views. Medin, Ross, and Markman say it most clearly when summarizing the research in the area of problem solving: "People are generally not very good at learning from the abstract descriptions of principles that are presented along with the illustrating problems. . . . Learning to solve problems appears to require the solution of many problems" (2001, 493). Gick and Holyoak (1980) found that they could increase the percentage of students who solved application-type problems from 10 percent to 75 percent simply by giving students direct hints to think about analogous problems that they had previously solved. In a later study, they found that giving students practice on more than one problem also improved performance

(Gick and Holyoak 1983). Cummins (1992) found that students were helped if they were asked to focus on understanding the underlying principles of the series of problems they were practicing. This improved their performance on test items even when they did not get the practice problems correct. The process of thinking about them was the key. It is clear that Arons's "exercises in verbal interpretation" encourage students to take this kind of approach to learning.

When we do not devote significant amounts of our time teaching students how to make these transformations, we must ask, "What are we really trying to do?" As mentioned earlier, one of my colleagues suggested that we are working a lot harder at demonstrating how clever we are than at helping students become competent in the field. Another colleague pointed out that when we weed out students, we are disproportionately removing students from underrepresented groups, including many students who are unable to afford high-priced college prep schools and who come from high schools where they do not have advanced classes. From this perspective, we are just part of the forces maintaining the advantages of the privileged. We may focus too much attention on evaluating and selecting students out of our disciplines and too little on helping each of them learn what our disciplines have to offer. In particular, we may be hurting the very students we say we want to help the most.

Key words: science, problem solving, knowledge transformations

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